C1 JAN 2011

1. (a) Find the value of $16^{-\frac{1}{4}}$ $= \frac{1}{2}$ **(2)**

(b) Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$ $= x(16x^{-1}) = 16$ **(2)**

2. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\,dx \quad = 2x^6 - x^3 + 3x^{\frac{4}{3}} + C$$

giving each term in its simplest form.

(5)

Leave blank

3. Simplify

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1} \quad \frac{\times\sqrt{3}+1}{\times\sqrt{3}+1} = \frac{-1+3\sqrt{3}}{2} = -\frac{1}{2}+\frac{3}{2}\sqrt{3}$$

giving your answer in the form $p+q\sqrt{3}$, where p and q are rational numbers.

(4)

4. A sequence a_1, a_2, a_3,... is defined by

$$a_1 = 2$$

$$a_{n+1} = 3a_n - c$$

where c is a constant.

$a_2 = 6 - c$

$a_3 = 18 - 3c - c$
$= 18 - 4c$

(a) Find an expression for a_2 in terms of c.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$ $\Rightarrow 2 + (6 - c) + (18 - 4c) = 0$

$26 = 5c$ $\quad c = \frac{26}{5}$

(b) find the value of c.

(4)

5.

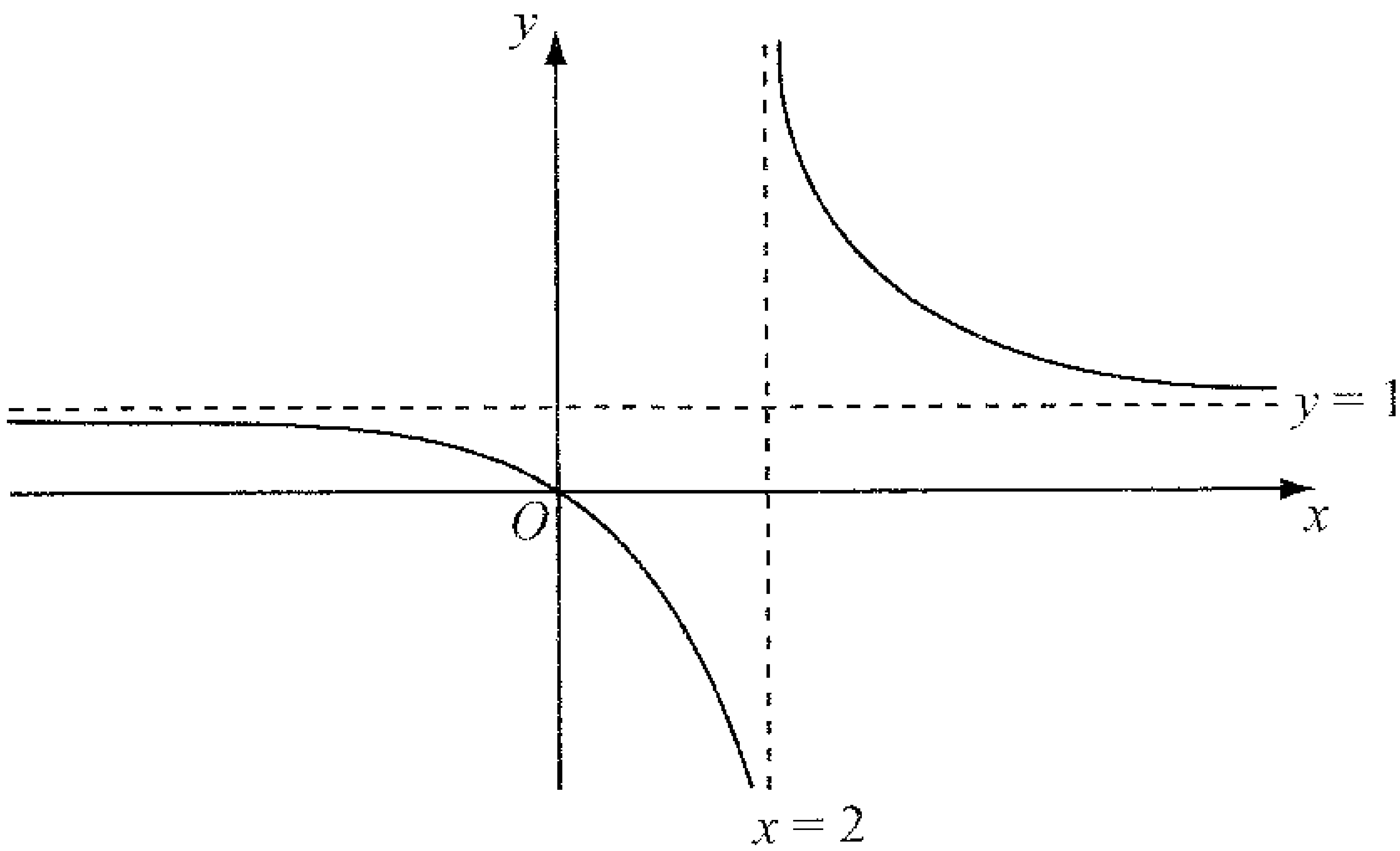


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2$$

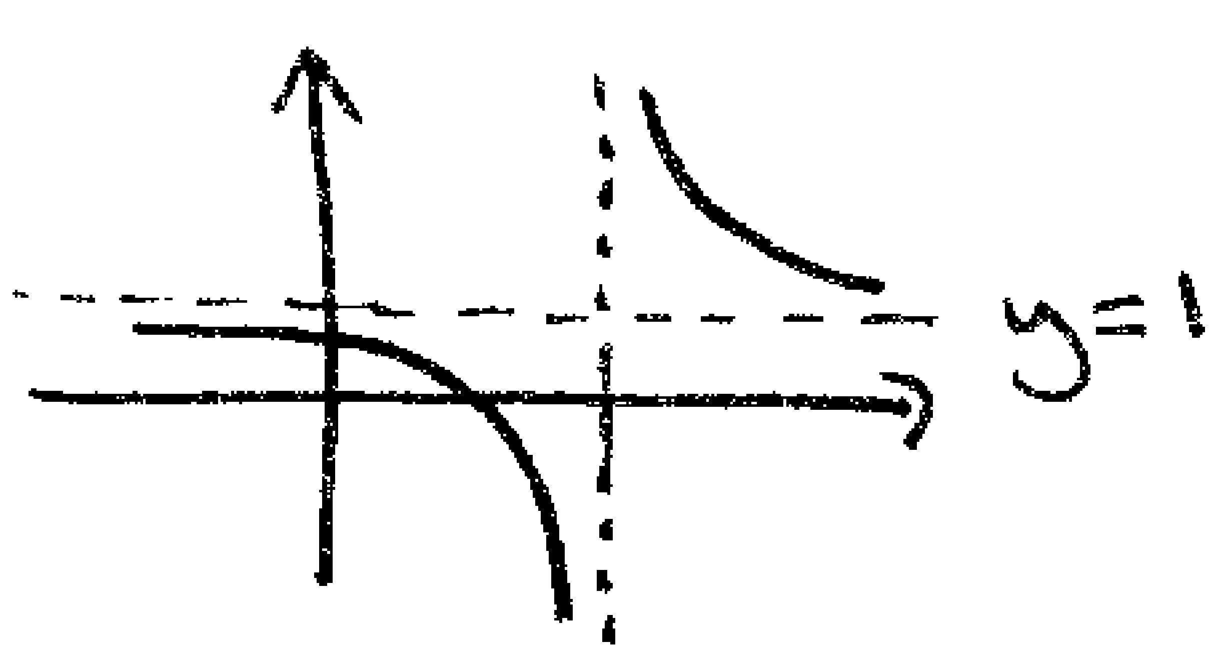
The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x-1)$ and state the equations of the asymptotes of this curve. $\xrightarrow[1]{}$

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x-1)$ crosses the coordinate axes.

(4)



asymptotes at $y=1, x=3$

b) $f(x-1) = \dfrac{(x-1)}{(x-1)-2} = \dfrac{x-1}{x-3}$

$x=0 \quad y = \dfrac{-1}{-3} \quad \underline{(0, \tfrac{1}{3})}$

$x=1 \quad y=0 \quad \underline{(1,0)}$

6. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

$S_{10} = 5(2a+9d) = 162$

(a) Show that $10a + 45d = 162$

$10a + 45d = 162$ **(2)**

Given also that the sixth term of the sequence is 17,

$a + 5d = 17$

(b) write down a second equation in a and d,

$\Rightarrow 10a + 50d = 170$ **(1)**

$-\ 10a + 45d = 162$

(c) find the value of a and the value of d.

$5d = 8$ **(4)**

$a + \cancel{5}\left(\frac{8}{\cancel{5}}\right) = 17 \quad \underline{a = 9}$

$d = \frac{8}{5}$

7. The curve with equation $y = f(x)$ passes through the point $(-1, 0)$.

Given that

$$f'(x) = 12x^2 - 8x + 1$$

find $f(x)$.

$f(x) = 4x^3 - 4x^2 + x + c$

$0 = -4 - 4 - 1 + c \Rightarrow c = 9$ **(5)**

$f(x) = 4x^3 - 4x^2 + x + 9$

$a=1 \quad b=k-3 \quad c=3-2k$

8. The equation $x^2+(k-3)x+(3-2k)=0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2+2k-3>0$$

(3)

$b^2-4ac>0$

$(k-3)^2-4(1)(3-2k)>0$

$k^2-6k+9-12+8k>0$

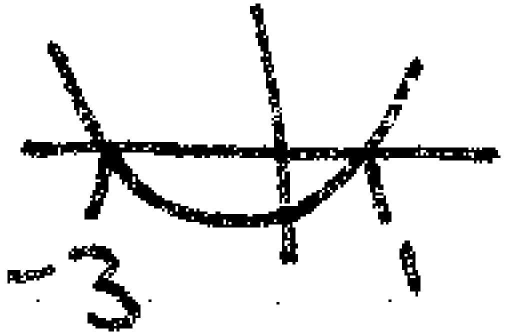
$k^2+2k-3>0$ #

(b) Find the set of possible values of k.

(4)

$(k+3)(k-1)>0$

$-3 \qquad 1$



$\Rightarrow \underline{k<-3} \quad \underline{k>1}$

9. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point A $(1, 4)$ lies on L_1, find $\quad 8-3-k=0 \Rightarrow \underline{k=5}$

(a) the value of k, $\quad 2y = 3x + 5 \Rightarrow y = \frac{3}{2}x + \frac{5}{2}$ **(1)**

(b) the gradient of L_1. $\quad m_{L_1} = \frac{3}{2}$ **(2)**

The line L_2 passes through A and is perpendicular to L_1. $\quad \Rightarrow m_{L_2} = -\frac{2}{3}$

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$y - 4 = -\frac{2}{3}(x-1) \Rightarrow 3y - 12 = -2x + 2$ **(4)**

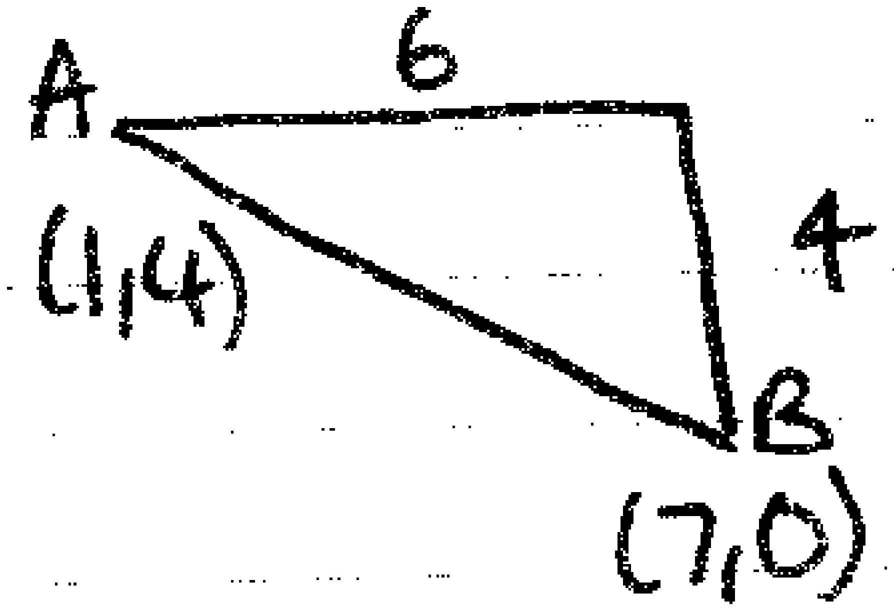
The line L_2 crosses the x-axis at the point B. $\quad \underline{2x + 3y - 14 = 0}$

(d) Find the coordinates of B.

$y = 0 \Rightarrow 2x = 14 \quad x = 7$ **(2)**

$\underline{B(7, 0)}$

(e) Find the exact length of AB. **(2)**

e) 

$AB = \sqrt{6^2 + 4^2} = \sqrt{52} = \underline{2\sqrt{13}}$

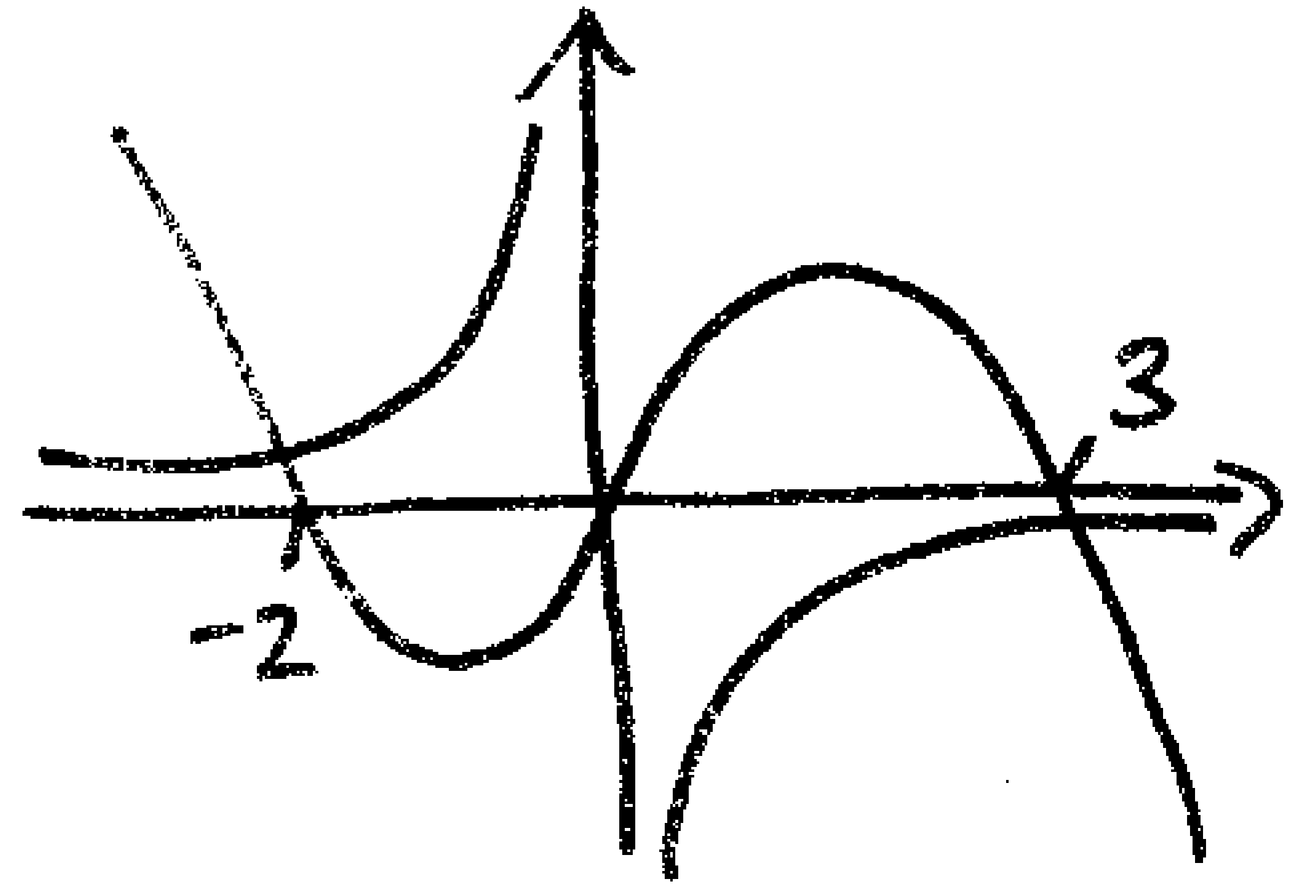
10. (a) On the axes below, sketch the graphs of

(i) $y = x(x+2)(3-x)$

0 -2 3

↓ hell

(ii) $y = -\dfrac{2}{x}$



showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0$$

(2)

graphs Intersect twice ⇒ 2 solutions

11. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0$$

(a) Find $\frac{dy}{dx}$. $= \frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$

(4)

(b) Show that the point $P(4, -8)$ lies on C. $y = \frac{1}{2}(4)^3 - 9(4)^{\frac{3}{2}} + \frac{8}{4} + 30$

(2)

$y = 32 - 72 + 2 + 30 = -8$ ✓

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

$m_t = \frac{3}{2}(4)^2 - \frac{27}{2}(4)^{\frac{1}{2}} - 8(4)^{-2} = 24 - 27 - \frac{1}{2} = -3\frac{1}{2} = -\frac{7}{2}$

$m_n = \frac{2}{7} \Rightarrow y + 8 = \frac{2}{7}(x - 4) \Rightarrow 7y + 56 = 2x - 8$

$2x - 7y - 64 = 0$